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Wavelet analyses and comparative denoised signals of meteorological factors of the namibian atmosphere

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ABSTRACT

With population growth and climate change increasing the exposure of communities and assets to extreme hydrological events, such as floods and droughts, it is crucial to make accurate and timely early warning, climate adaptation and water management information available that can help minimize the loss of the limited quantities of water in arid regions. Trend estimation using wavelets has continued to attract ubiquitous applications in various fields including atmospheric and water studies. While Fourier analysis employs big waves, wavelet analysis uses small waves. Since wavelets localise features in a signal data to different scales, we can therefore preserve vital signal features while removing noise in the signal. Hence the basic notion of wavelet denoising or wavelet thresholding is that the wavelet transform results in a sparse representation for many real-world signals, meaning that the wavelet transform concentrates signal features in a few large-magnitude wavelet coefficients. Since small wavelet coefficients are typically noise, we can "shrink" or remove them without affecting the signal quality.

This paper employs wavelet analyses of some selected meteorological factors of the Namibian atmosphere to investigate the noise in the data collected from a number of Southern African Science Service Centre for Climate Change and Adaptive Land Management (SASSCAL) stations, for the years 2012 to 2015. The results show some significant levels of noise around the thresholds of various optimal time series peaks of the data. Hence the denoised data serve to represent fairly refined representations of data for the given period. Moreover, the results of comparing the optimal game model solutions for the original and denoised data specifically identified some weather stations that are "humidity noise stable" and "temperature noise stable". Thus, these suggest that despite the noise in the respective meteorological data for these stations, the effects of the noise in the data appear to be manageable or optimally controllable, within the given period.

1. Introduction

Namibia is characterised by a complex earth-atmospheric interactions system of high temperature, low relative humidity, high evaporation, and evapo-transpiration inland; low precipitation, low temperatures, and moist air at the Atlantic coast (Namib Desert); high temperatures and frequent floods in the north-east; and high temperatures and alternating floods and droughts in the central-north (Kgabi et al., 2016). The situation demands accurate hydro-climatic data (including meteorological parameters) for understanding of the atmospheric water-climate interactions, and for effective forecasting, monitoring and preservation of the water resources.

Wavelet transform has been used in several predictive water quality modelling studies including Dökmen and Aslan (2013), to obtain water time sequence variations in different scale, and to improve accuracy of the prediction models. Zhang et al. (2016) combined the wavelet transform and the chaotic prediction model, which has simple calculations and high precision to prove the validity of the new model. In this study, the wavelet transform is used to conduct wavelet analyses of selected precipitation-enhancing meteorological parameters (temperature, relative humidity, wind speed) of the Namibian atmosphere, in an attempt to investigate the noise in the data collected from a number of SASSCAL stations (See Fig. 2), for a four-year (2012 to 2015) period. The ultimate objective of the study is thus to improve the quality of the limited hydro-climatic data, and to provide significant early warning and decision support for water quantity and quality protection in arid environments.

The wavelet transform (WT) is a mathematical tool extensively

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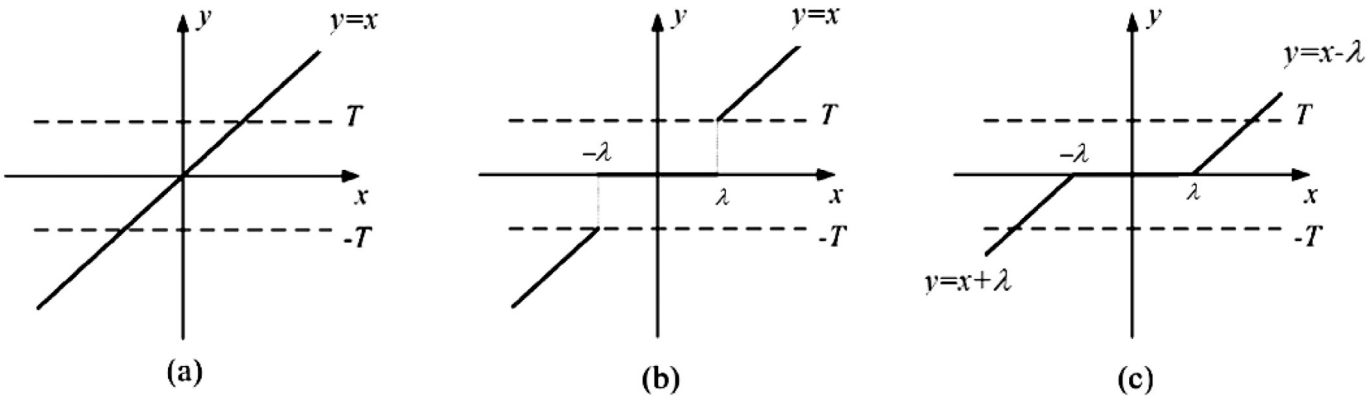


Fig. 1. Thresholding function: (a) original signal (b) hard thresholding (c) soft thresholding.

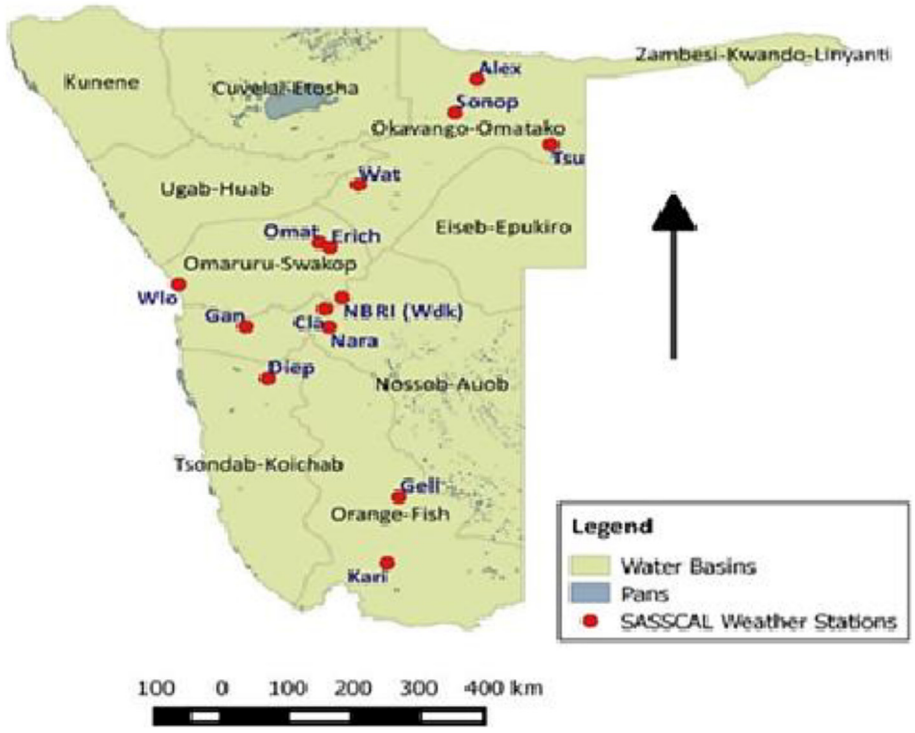


Fig. 2. Map of the Selected SASSCAL Weather Stations.

useful in the analysis of signals whose representation involves the decomposition of the signals in wavelet basis functions ψ given by

$$\psi_{(a,b)}(t) = (1/\sqrt{a})\psi((t-b)/a) \quad (1)$$

where a is a scale and b is a translation.

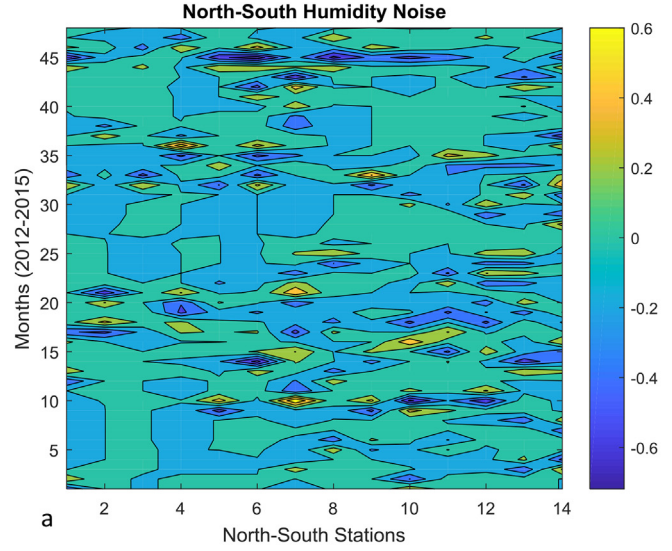
The continuous wavelet transform (CWT) can be thought of as the inner product in $L^2[a,b]$ of the test signal $f(t)$ with the basis functions $\psi_{a,b}(t)$:

$$\Psi_f^{\psi}(a,b) = C(a,b) = \int_{-\infty}^{\infty} f(t)\psi_{a,b}(t)dt \quad (2)$$

Generally, wavelet thresholding for a signal (or an image) denoising is aimed at removing the noise present in the signal while preserving most of the signal properties, irrespective of its frequency content. The steps involved in thresholding (Taswell, 1999; Shim et al., 2001; Zhang et al., 2007) are (i) acquiring the noisy digital signal; (ii) computing a linear forward discrete wavelet transform of the noisy signal; (iii) performing a non-linear thresholding operation on the wavelet coefficients of the noisy signal, and (iv) computing the linear inverse wavelet transform of the thresholded wavelet coefficients.

Threshold is the estimated noise level with the values larger than the threshold being regarded as signal, while the smaller ones are regarded as noises.

The above simple four-step process is known as wavelet thresholding or shrinkage.



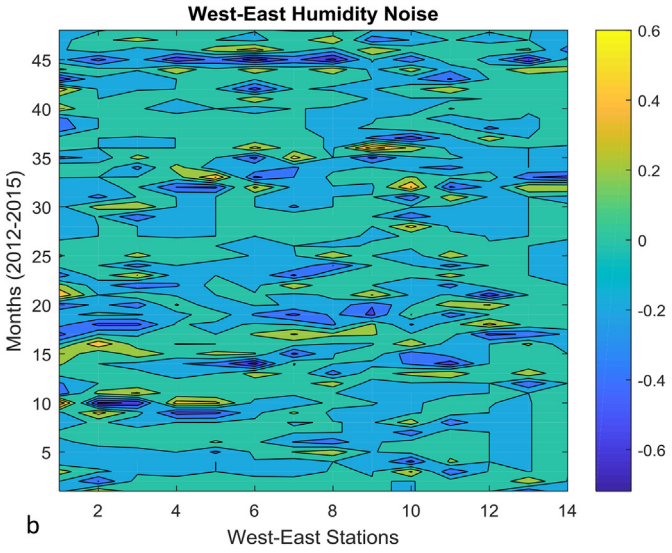
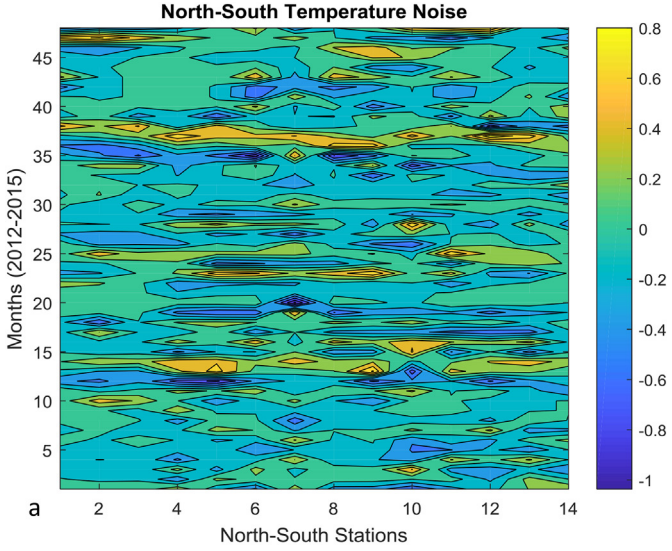


Fig. 3. a. North-South Humidity Noise Profile. b. West-East Humidity Noise Profile.



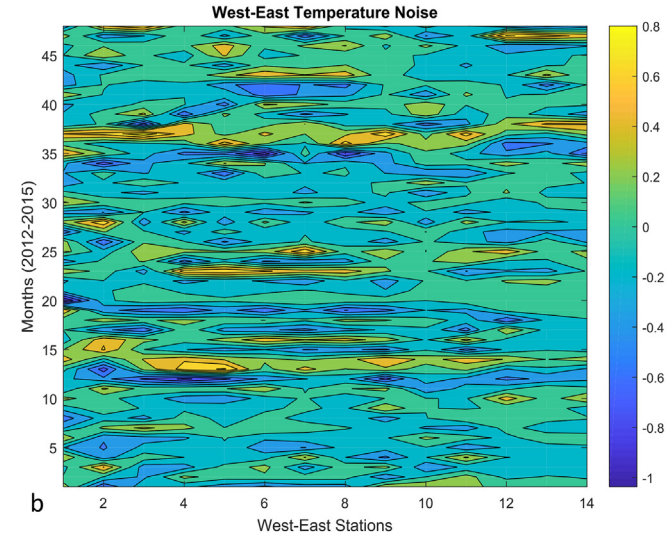


Fig. 4. a. North-South Temperature Noise Profile. b. West-East Temperature Noise Profile.

Assume the noisy (original) signal $y = (y_i)_{i\in[1,N]}$ is given by

$$y = x + n \tag{3}$$

where $x = (x_i)_{i\in[1,N]}$ is the noise-free signal and $n = (n_i)_{i\in[1,N]}$ is Gaussian white noise with independent identical distributions $N(0, {\sigma_n}^2)$.

In the wavelet domain, the noisy signal can be represented by

$$w = u + v \tag{4}$$

where $w = (w_i)_{i\in[1,N]}$ is the noisy wavelet coefficients vector, and $u = (u_i)_{i\in[1,N]}$ represents the noise-free wavelet coefficients vector corrupted by the noise coefficients vector $v = (v_i)_{i\in[1,N]}$.

The goal of noise reduction is to obtain an estimate $\hat{u}$ of the noise-free wavelet coefficients vector u by using a wavelet shrinkage function. Two most popular shrinkage functions are the hard and soft shrinkage functions (Pizurica and Philips, 2004). They are the standard thresholding schemes or policies (Pizurica et al., 2002), also termed "keep or kill" (i.e. hard-thresholding) and "shrink or kill" (i.e. soft-thresholding). The hard shrinkage function (HSF) is defined as

$$T_h(w_i, \lambda) = \begin{cases} w_i, & \text{if } |w_i| \geq \lambda \\ 0, & \text{otherwise} \end{cases} \tag{5}$$

The soft shrinkage function (SSF) is defined as

$$T_s(w_i, \lambda) = \begin{cases} (|w_i| - \lambda), & \text{if } |w_i| \geq \lambda \\ (|w_i| + \lambda), & \text{if } |w_i| \leq -\lambda \\ 0, & \text{otherwise} \end{cases} \tag{6}$$

where $\lambda = \sigma\sqrt{2logN}$.

Obviously, as $N \to \infty$, the threshold λ increases, and as $N \to 0$, the threshold λ decreases, which is undesirable for the hard and the soft

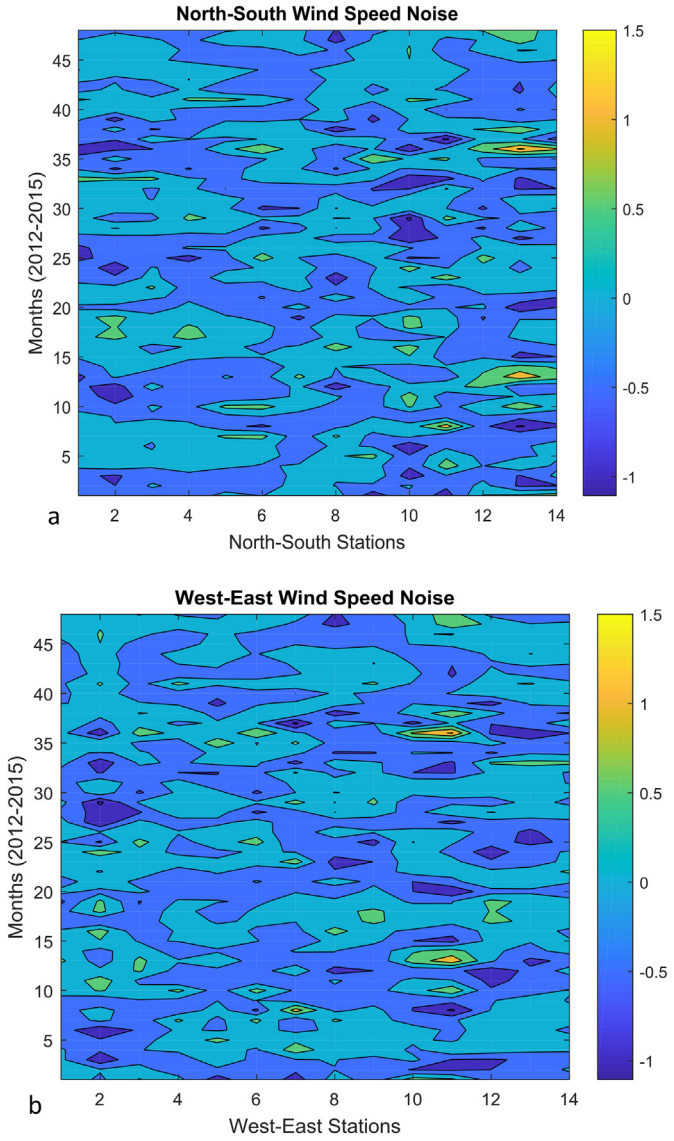


Fig. 5. a. North-South Wind Speed Noise Profile. b. West-East Wind Speed Noise Profile.

shrinkage functions.

Wavelet analysis has developed in the past 20 years as an analytic method and can be used to make up for the defects of the discrete Fourier transform with a fixed window and no discrete orthogonal basis (Zhang et al., 2016).

used in this paper are the Minimax Criterion technique (Donoho and Johnstone, 1998) and Stein's unbiased risk estimator (SURE) (Stein, 1981). The former employs the statistical Minimax principle that uses a fixed threshold λ to yield minimax performance for mean square error against an ideal procedure. Using the basic objective of data assimilation, viz., the extraction of a signal from noisy observations, the denoised signal can be assimilated to the estimator of an unknown regression function, hence the minimax estimator is an option that realises the minimum over a given set of functions of the maximum mean square error (MSE), and the procedure finds optimal thresholds. On the other hand, the soft threshold rigorous SURE is defined as follows:

Consider a vector $W = [w_1, w_1, \ldots, w_N]$, consisting of the square of wavelet coefficients from small to large. We select the minimum value r_b as the risk value from risk vector, which is given as

$$R = \{r_i\}_{i=1,2,\ldots,N} = \frac{[N - 2i + (N - i)w_i + \sum_{k=1}^{l} w_k]}{N}$$

The selected threshold is $= \sigma\sqrt{w_b}$, where w_b is the bth squared wavelet coefficient (at minimum risk) chosen from the vector W and σ is the standard deviation of the noisy signal.

2. Denoising modelling procedure

A more precise mathematical formulation of the general wavelet denoising procedure employs the following variables' definitions:

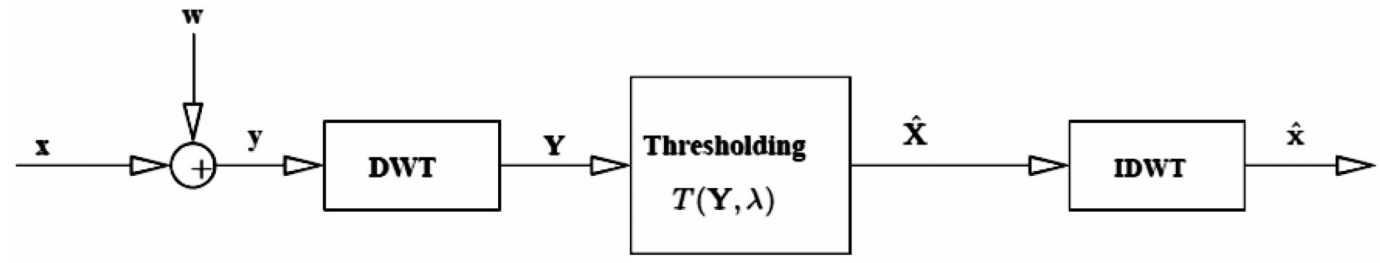
Let

i. $x = [x_i]$, $i = 1, 2, \ldots, M$, be the original noise-free signal having M samples.
ii. $X = DWT(x)$, be the discrete wavelet transform of the original signal x.
iii. w, be an additive Gaussian white noise with zero mean and variance σ_w^2 which is assumed to have the same size M but independent of the original signal x, so $w \sim N(0, \sigma_w^2)$.
iv. $y = x + w = [y_i]$, $i = 1, 2, \ldots, M$, the noisy version of the original noise-free signal.
v. $Y = DWT(y)$, be the discrete wavelet transform of the noisy signal y.
vi. $\hat{X} = T(X, \lambda)$, the thresholded wavelet coefficients obtained from the thresholding operation defined by Eq.(3) or (4).
vii. $\hat{x} = DWT^{-1}(\hat{X})$, the denoised version of the noisy signal y, which represents an approximation of the original signal x, with mean squared error:

$$MSE = E[\|x - \hat{x}\|^2]$$

which is to be minimised.

Thus, in summary, the wavelet denoising procedure is illustrated by the following block diagram



1.1. Selected examples of thresholding techniques

There are various shrinkage techniques but two common examples where the transfer or shrinkage functions $T_{hard}(w_i, \lambda)$ and $T_{soft}(w_i, \lambda)$ are as shown in the following illustrative figures adapted from Luo and Zhang (2012) (See Fig. 1).

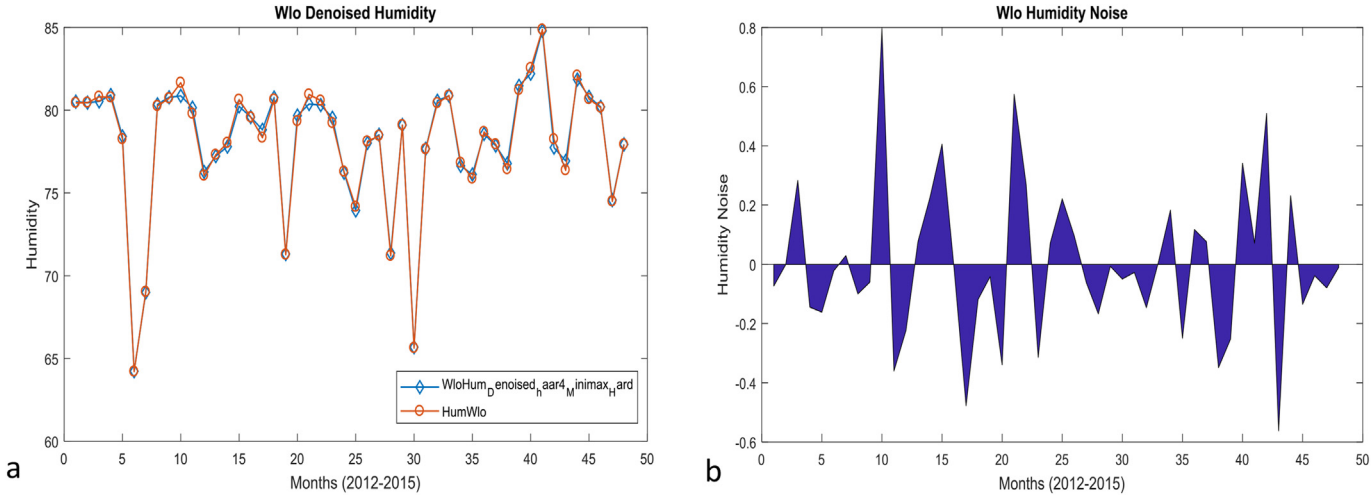


Fig. 6. a. WLO Station Original & Denoised Humidity. b. WLO Station Humidity Noise Density.

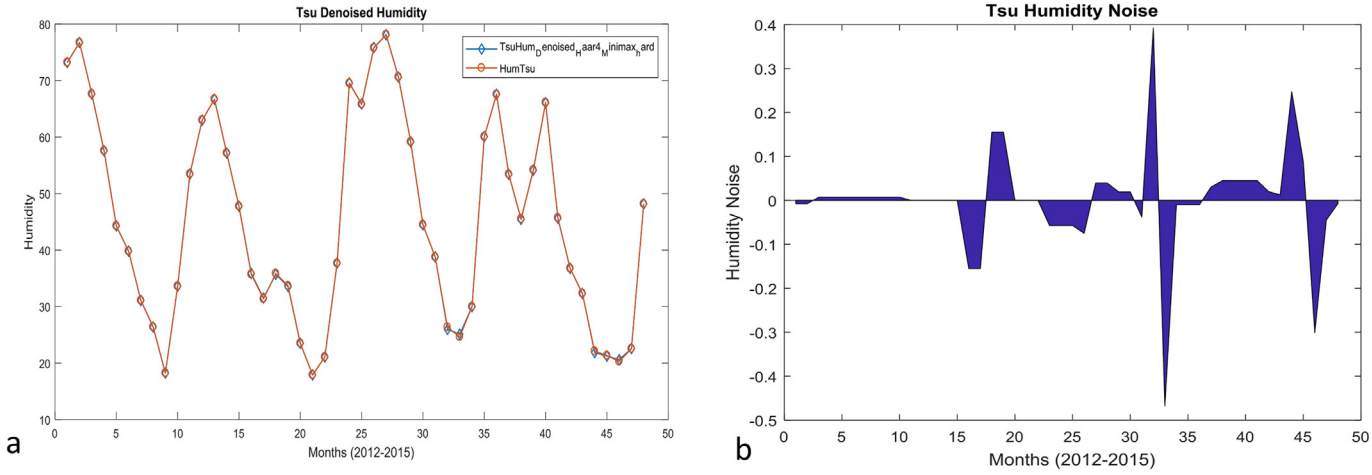


Fig. 7. a. TSU Station Original & Denoised Humidity. b. TSU Station Humidity Noise Density.

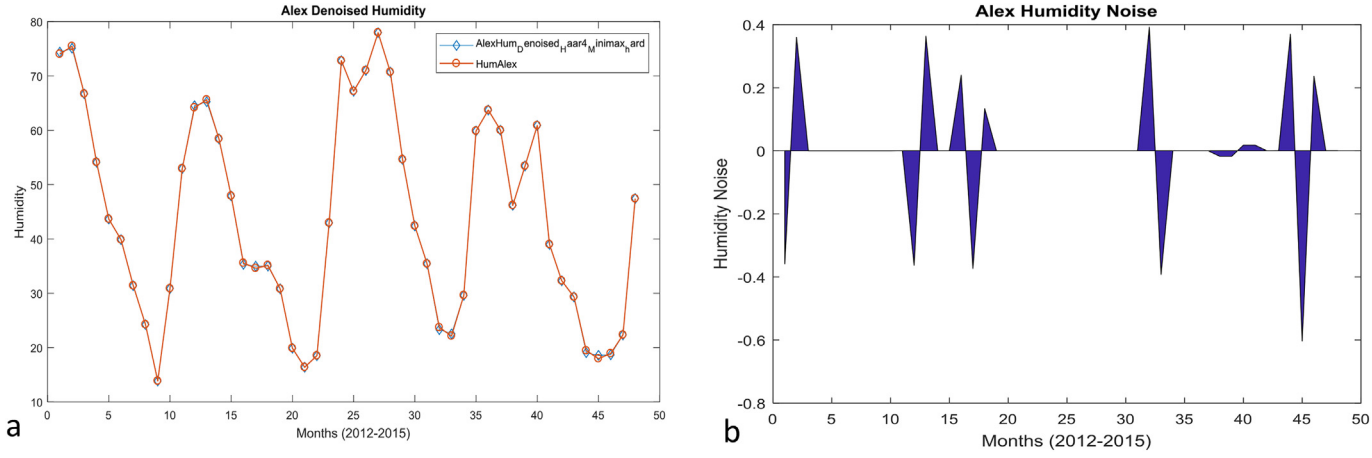


Fig. 8. a. ALEX Station Original & Denoised Humidity. b. ALEX Station Humidity Noise Density.

The hard thresholding, especially, as a “keep or set to zero” procedure is more intuitively appealing (Ghazel, 2004). On the other hand, soft thresholding shrinks coefficients above the threshold in absolute value. While at first sight hard thresholding may seem to be natural, the continuity of soft thresholding has some advantages. Sometimes, pure noise coefficients may pass the hard thresholding and appear as irritating “blips” in the output. However, soft thresholding shrinks these false structures. After the thresholding operator T(.) has been defined, the problem remains that of selecting the corresponding threshold.

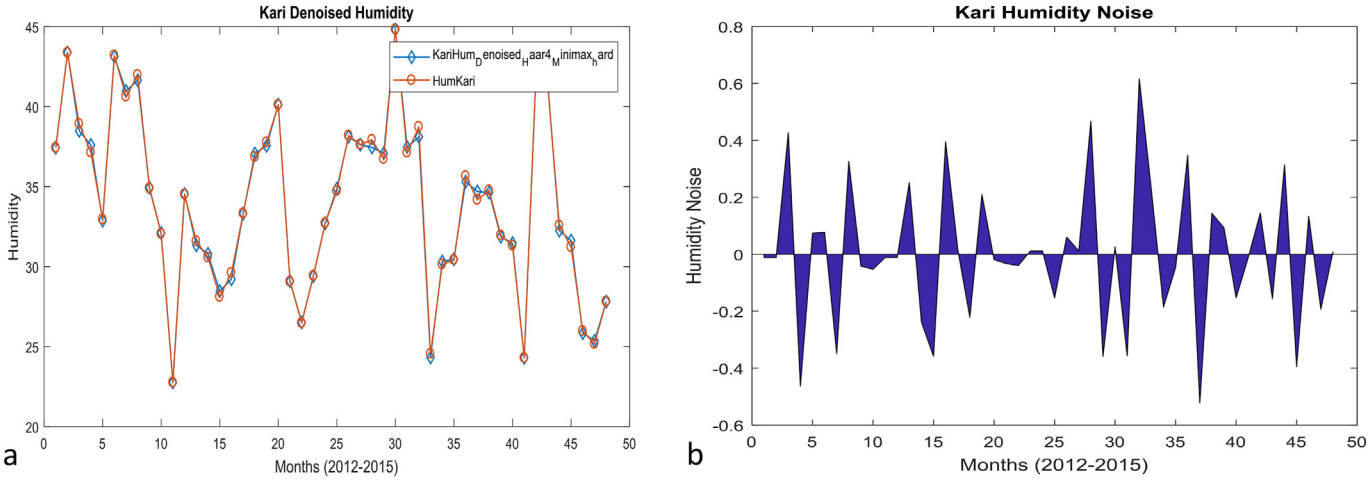


Fig. 9. a. KARI Station Original & Denoised Humidity. b. KARI Station Humidity Noise Density.

2.1. Computed estimated denoised data of meteorological factors of the namibian atmosphere

Using a mixed strategy game modelling approach for obtaining the optimal meteorological factor values defined by relative humidity, temperature, wind speed and leaf wetness data obtained from 14 Southern African Science Service Centre for Climate Change and Adaptive Land Management (SASSCAL) Namibian stations (See Fig. 2), for the years 2012 to 2015 (made up of 48 months), we investigate the noise in the data collected from the same weather stations over the same period, employing the Haar wavelet of level 4 (for its simplicity among the several typical wavelet transforms) with hard thresholding Minimax and Rigorous SURE procedures to obtain the following denoised data for the meteorological factors, as shown in Figs. 3a–17b. The Rigorous SURE was specifically employed for the wind speed due to the peculiarity of the data, especially the relatively less noisy observations unlike other meteorological factors. Hence the wind speed data denoising requires an optimum strategy such as the thresholding

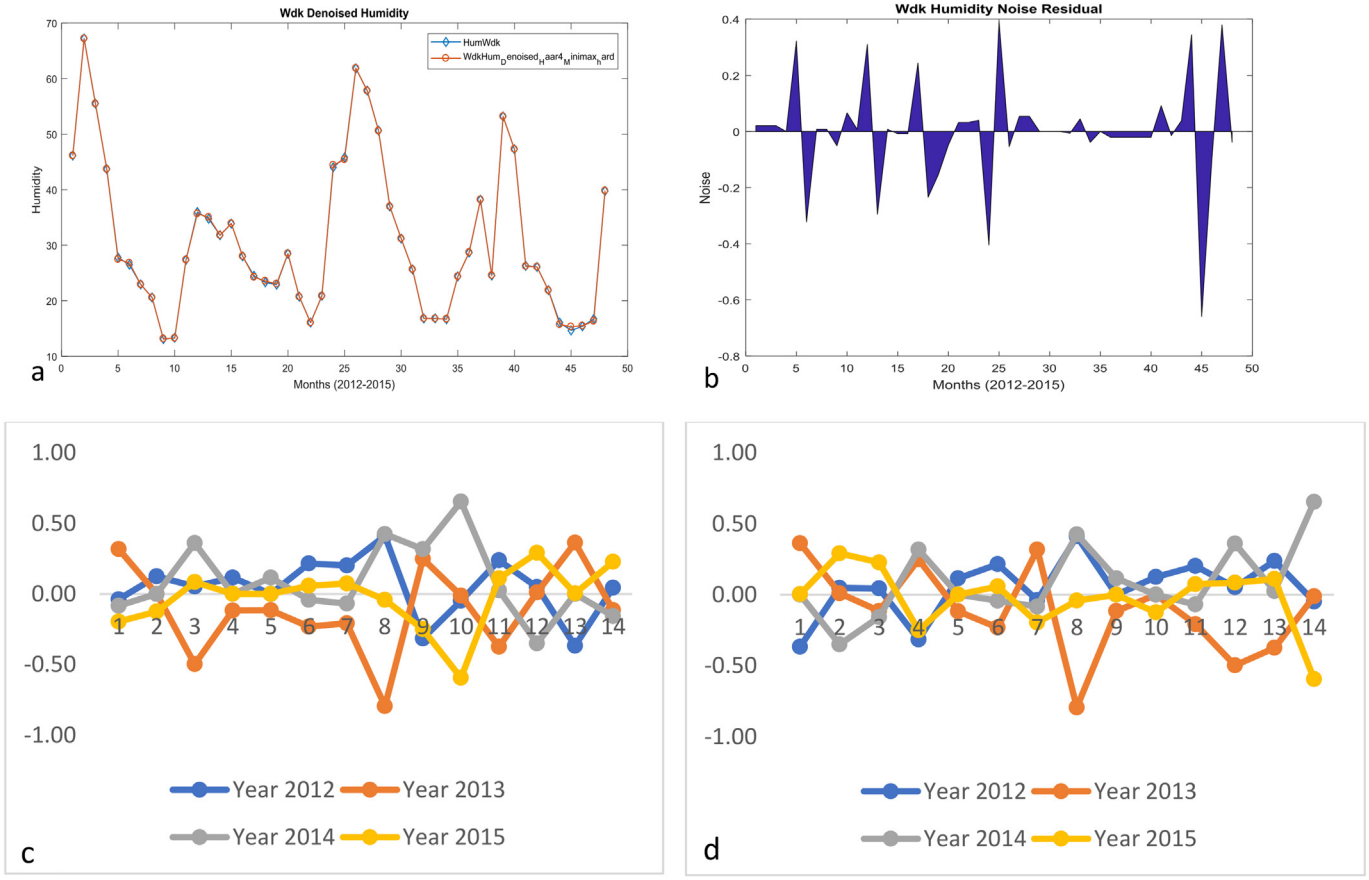


Fig. 10. a. WDK Station Original & Denoised Humidity. b. WDK Station Humidity Noise Density. c. Annual (West-East) Humidity Noise Residuals. d. Annual (North-South) Humidity Noise Residuals.

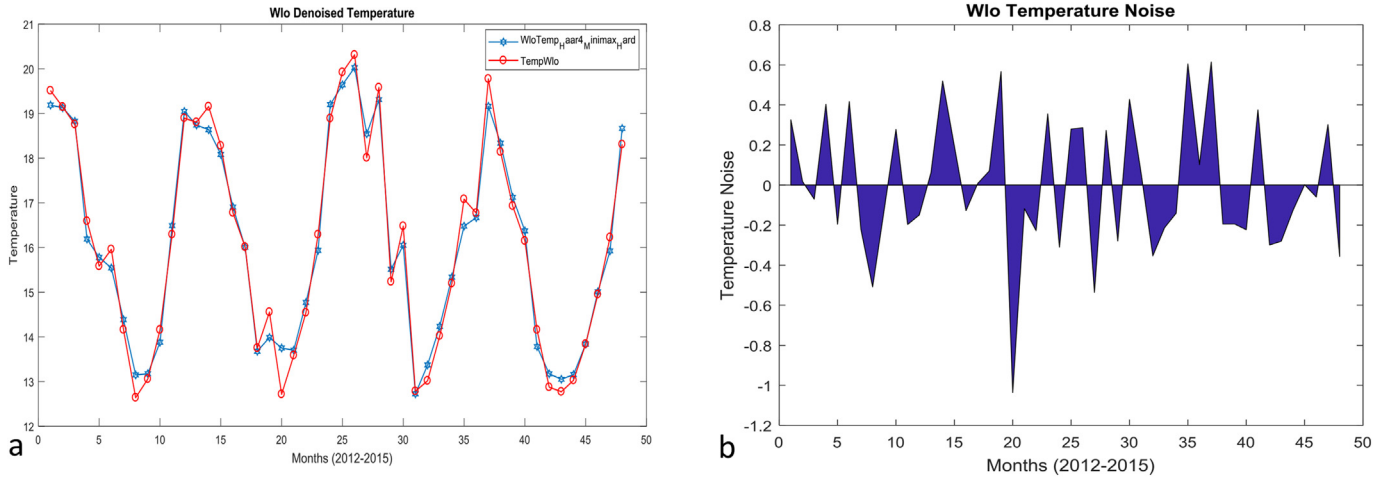


Fig. 11. a. WLO Station Original & Denoised Temperature. b. WLO Station Temperature Noise Density.

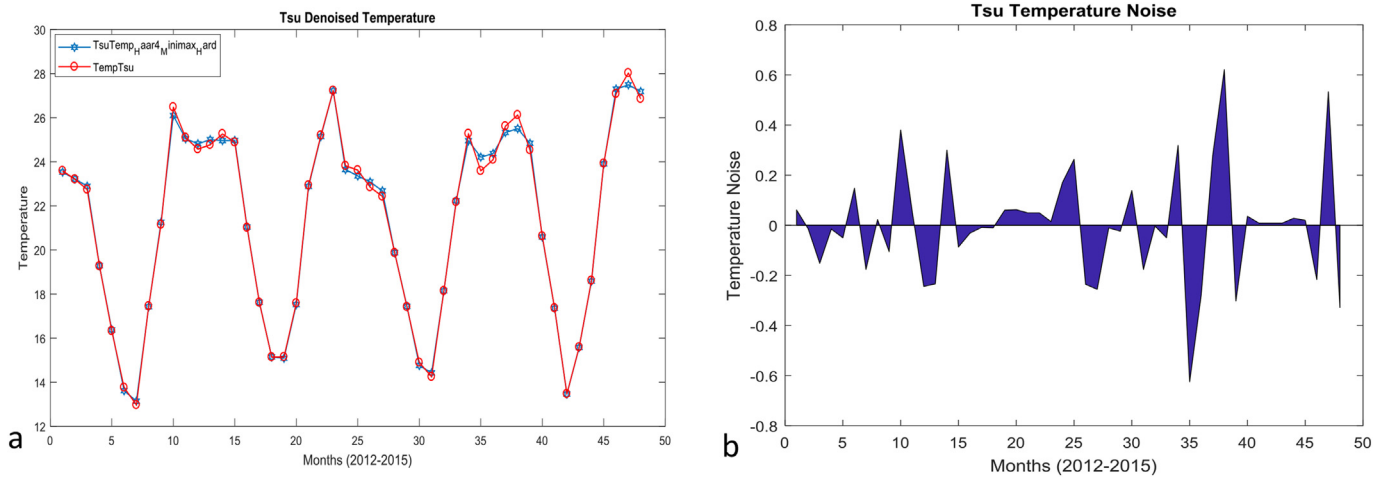


Fig. 12. a. TSU Station Original & Denoised Temperature. b. TSU Station Temperature Noise Density.

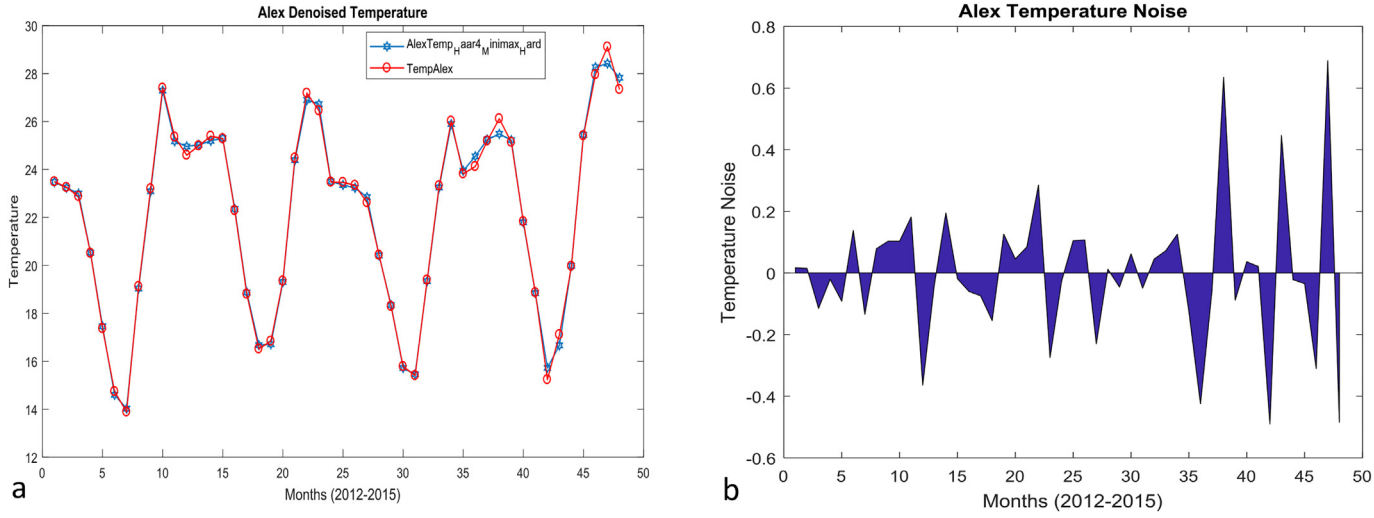


Fig. 13. a. ALEX Station Original & Denoised Temperature. b. ALEX Station Temperature Noise Density.

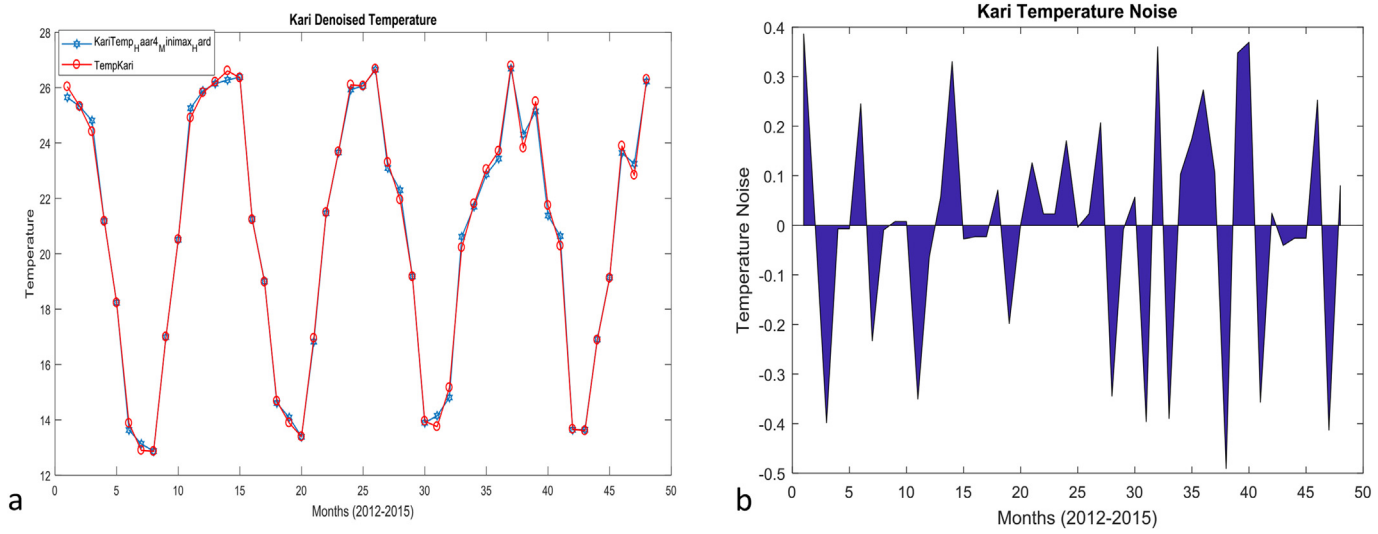


Fig. 14. a. KARI Station Original & Denoised Temperature. b. KARI Station Temperature Noise Density.

method of heuristic SURE-hard, for adequate noise removal as used by (Cao et al., 2003) in their signal denoising work.

The plots showing the meteorological factor noise distributions or profiles for the stations during the given period when observed from West to East and North to South of the country are shown in Figs. 3–5.

The West-East and North-South arrangements of the weather stations are as follows:

⟨West → East⟩:

{1. Wlo 2. Gan 3. Diep 4. Omat 5. Cla 6. Erich 7. Nara 8. Wdk 9. Wat 10. Kari 11. Gell 12. Sonop 13. Alex 14. Tsu}

⟨North → South⟩:

{1.Alex 2.Sonop 3. Tsu 4. Wat 5.Omat 6.Erich 7. Wlo 8. Wdk 9. Cla 10. Gan 11.Nara 12.Diep 13.Gell 14.Kari}

From the humidity noise profiles in Fig. 3a and b above, it is more evident that 2012 was less noisy than other years, in both cardinal orientations. Moreover, the southern part of the country was more humidity noisy than the north while the extreme western part of the country was more noisy than the eastern part. In general there were significant noise bursts scattered across the central region of the country.

The temperature noise profiles in Fig. 4a and b above reveal again that 2012 was less temperature noisy than the subsequent years. Specifically, 2013 and 2015 seemed to be significantly noisy across the stations in both cardinal directions.

The wind speed data across the stations and over the given period were noticeably less noisy but characterised by very few significant noise bursts toward the south and east of the country.

In the following figures (Fig. 6a–20b), we have the comparison plots for the original (S) and the denoised data (Ds), alongside the noise densities for the selected stations representing the West, East, North, South and Central (and specifically the national capital) meteorological environments. The horizontal axis for the plots is calibrated for the 48 months of the four year-period (2012–2015), shown numerically to avoid very crowded labels. Figs. 6-20.

Table 1
Densities and spread of humidity and temperature noises.

	Widely spread noise pops		Fairly/less spread noise pops	
	Humidity denoised data	Temperature denoised data	Humidity denoised data	Temperature denoised data
Higher noise densities	WLO (West) KARI (South)	ALEX (North) WLO (West)		
Lower noise densities	WDK (Central)	KARI (South) WDK (Central)	ALEX (North) TSU (East) WDK (Central)	TSU (East)

Table 2
Density and spread of wind speed noise.

	Widely spread noise pops	Fairly/less spread noise pops
Higher noise densities	KARI (South) TSU (East) WDK (Central)	
Lower noise densities		ALEX (North) WLO (West)

2.2. Summarised results for humidity denoised data

i. The Western and the Southern parts of the country appear to be more humidity noisy with higher values as observed from the WLO and KARI noise densities (See Table 1).
ii. The Eastern and Northern parts of the country appear to be less and sparsely humidity noisy with low values as observed from the TSU and ALEX noise densities (See Table 1).
iii. The central part depicts low noise density akin to the Northern and Eastern parts with fewer noise pops than the Western and Southern stations (See Table 1).
iv. The annual humidity noise residual plots reveal a noisier central part of the country with reduced humidity noise farther east unlike farther south.

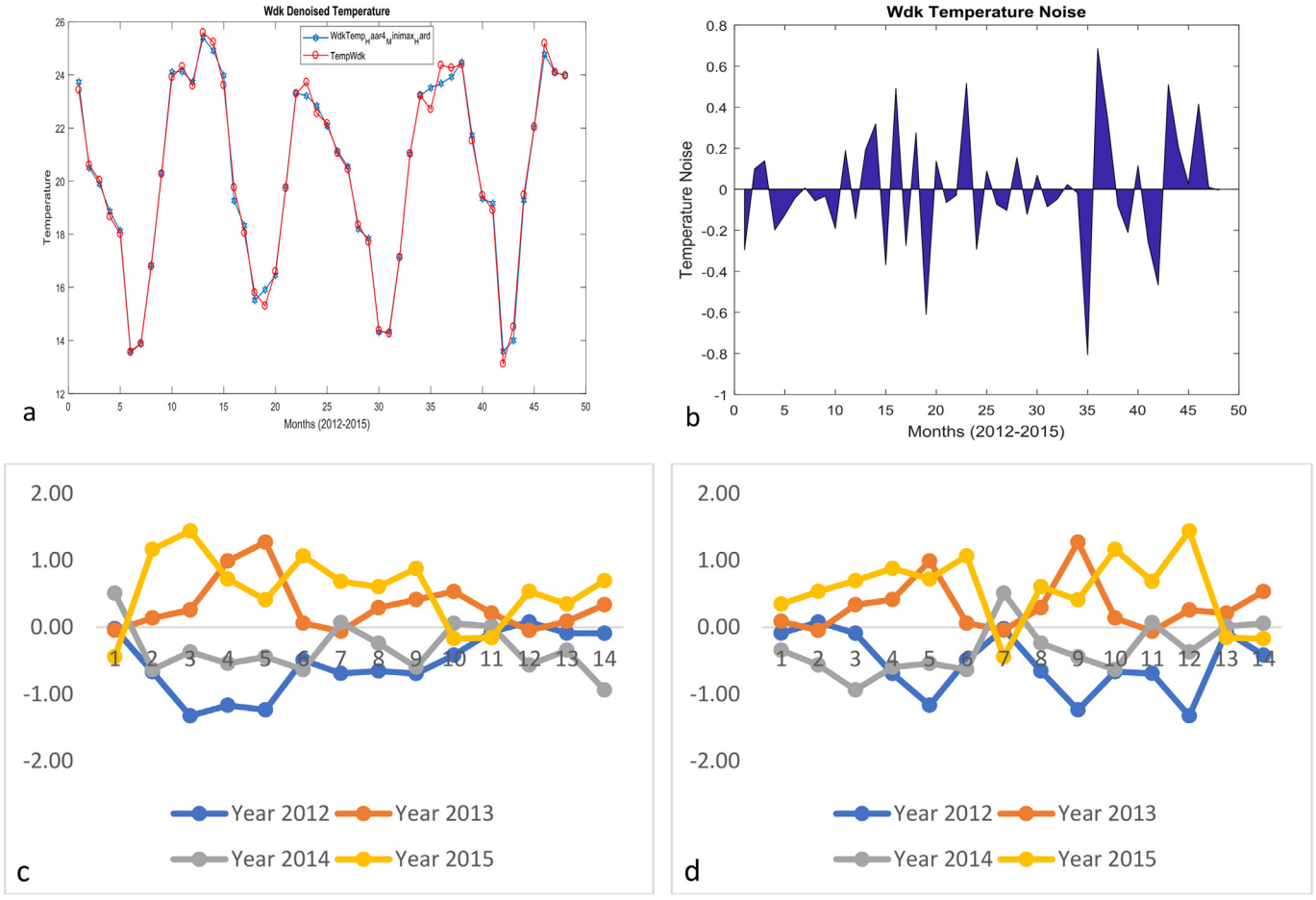


Fig. 15. a. WDK Station Original & Denoised Temperature. b. WDK Station Temperature Noise Density. c. Annual (West-East) Temperature Noise Residuals Fig. 15d: Annual (North-South) Temperature Noise Residuals.

2.3. Summarised results for temperature denoised data

i. Western station of WLO has higher with widely spread noise pops over the period (See Table 1).
ii. Central station of WDK has widely spread pops over the period with just very few higher densities (See Table 1).
iii. Southern station of KARI depicts widely spread noise pops but fairly with lower densities (See Table 1).
iv. Northern station of Alex with widely spread a few higher densities around 2014–2015 (See Table 1).

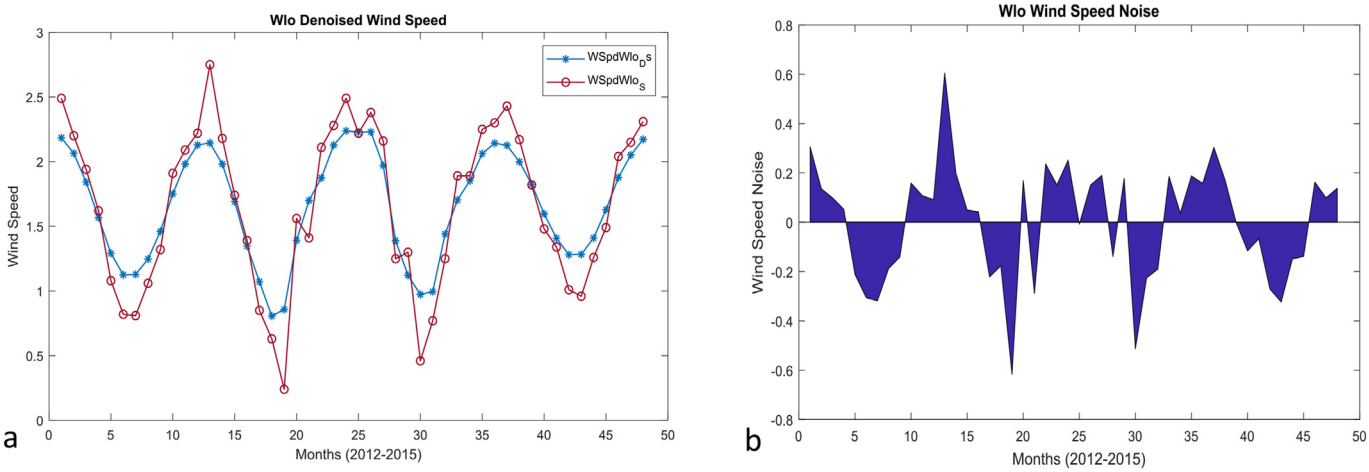


Fig. 16. a. WLO Station Original & Denoised Wind Speed. b. WLO Station Wind Speed Noise Density.

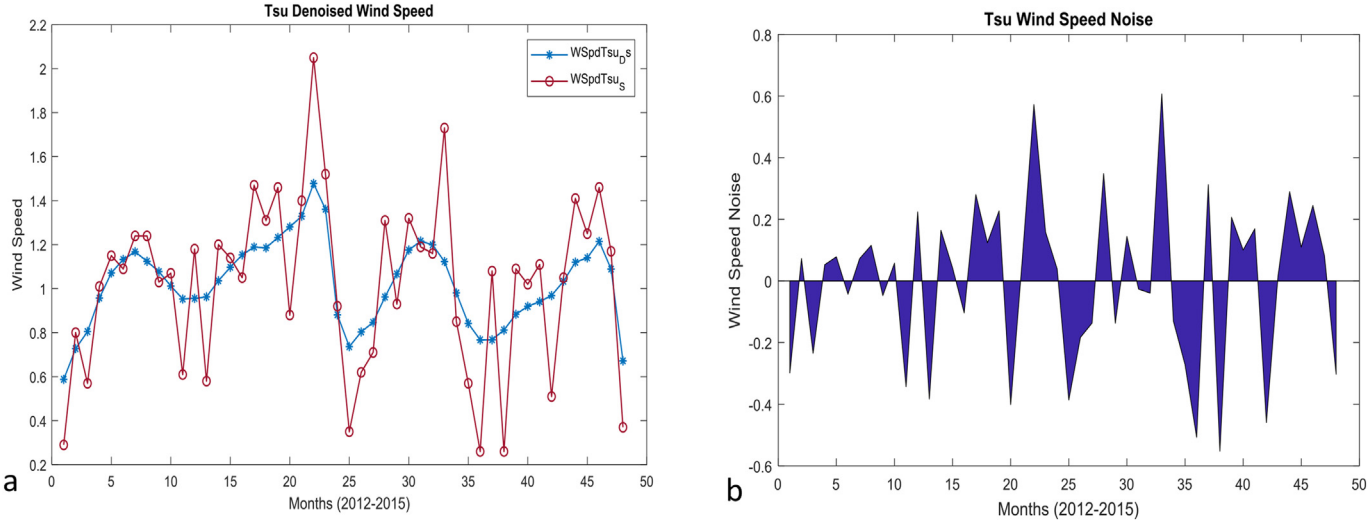


Fig. 17. a. TSU Station Original & Denoised Wind Speed. b. TSU Station Wind Speed Noise Density.

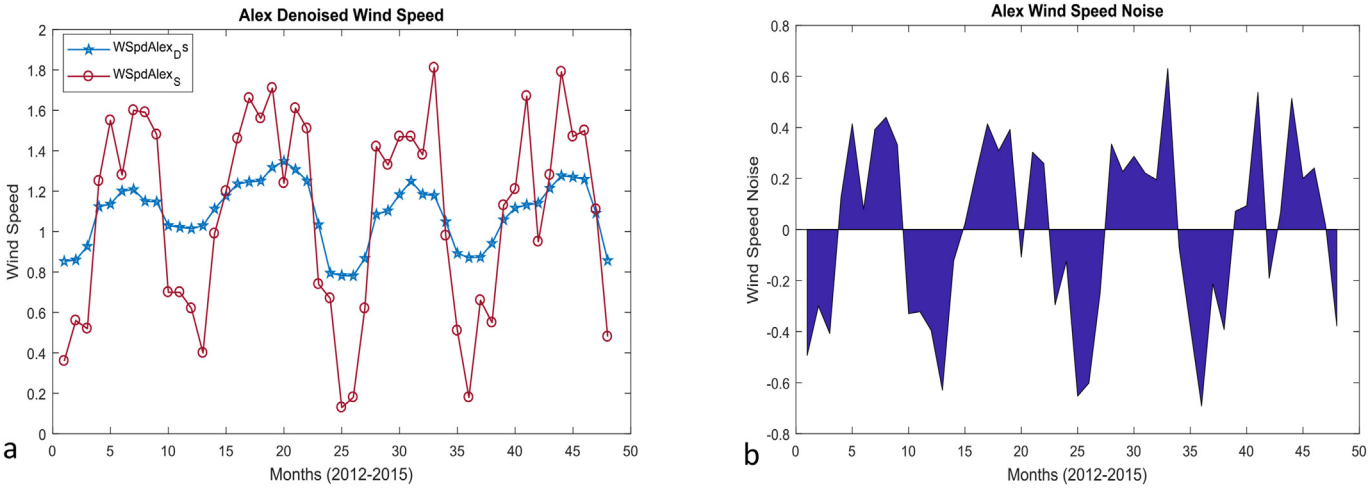


Fig. 18. a. ALEX Station Original & Denoised Wind Speed. b. ALEX Station Wind Speed Noise Density.

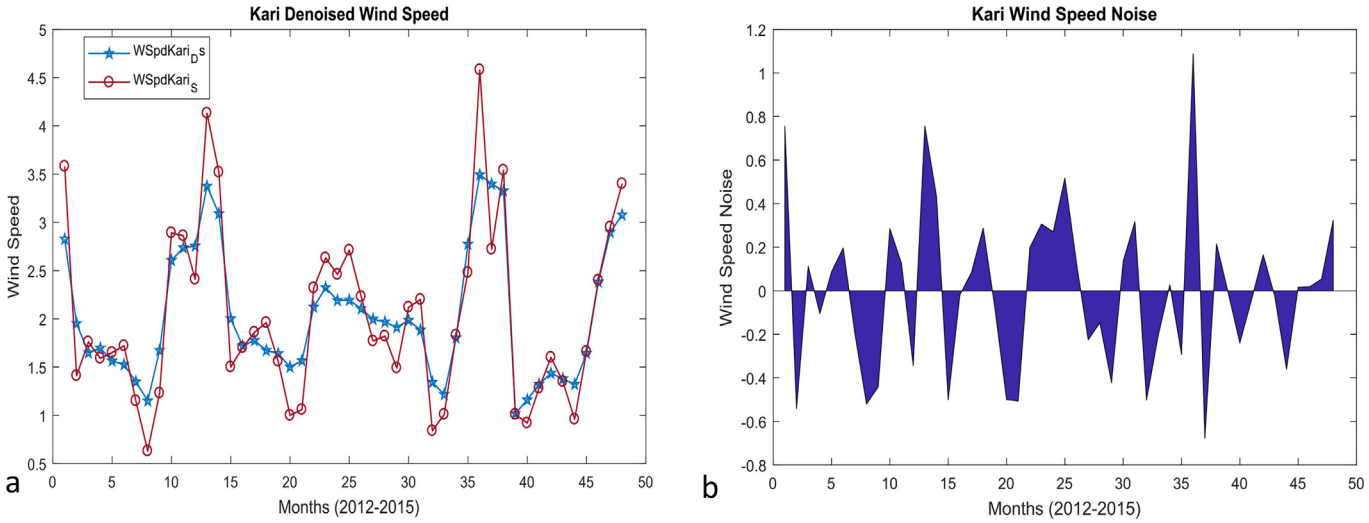


Fig. 19. a. KARI Station Original & Denoised Wind Speed. b. KARI Station Wind Speed Noise Density.

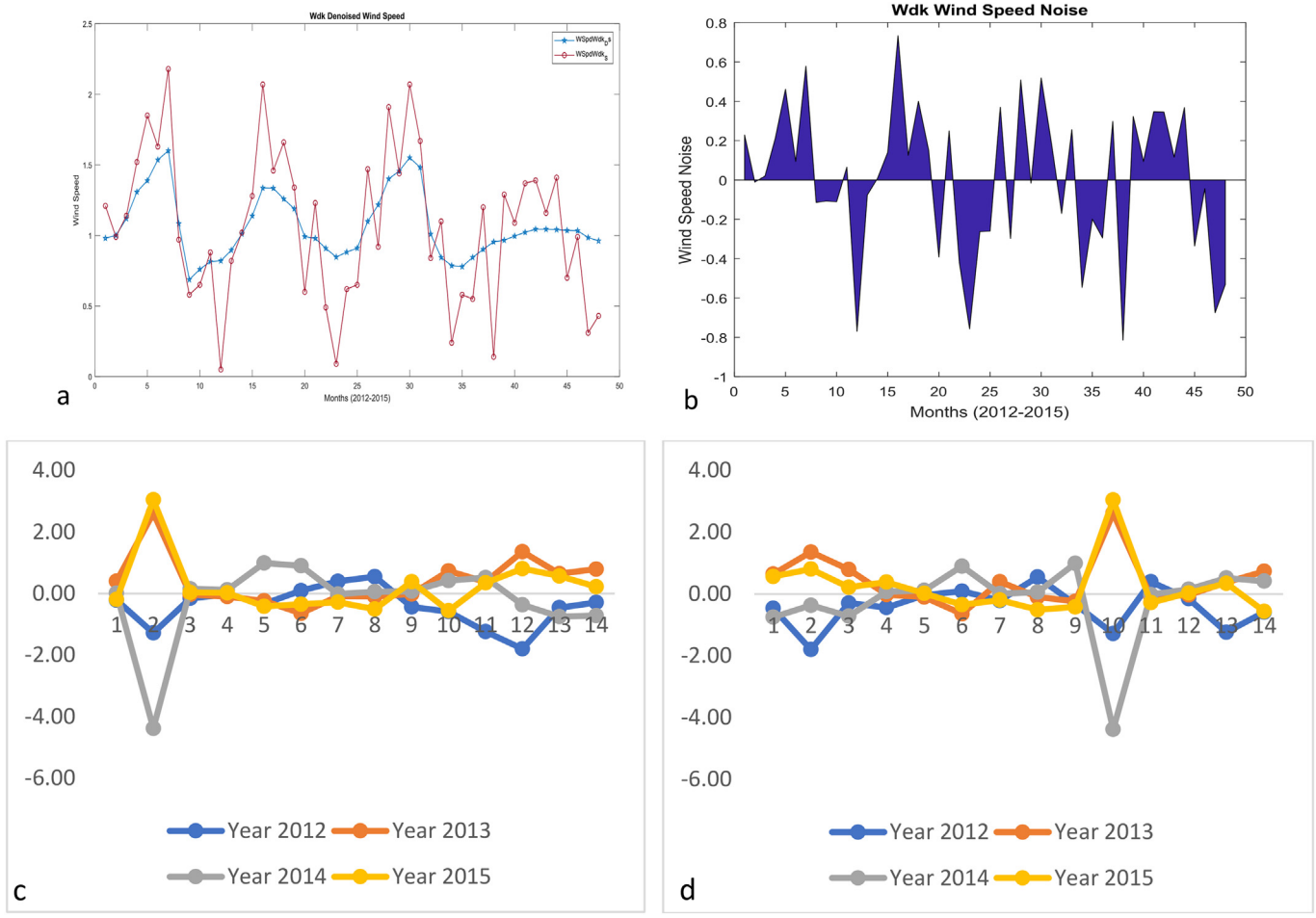


Fig. 20. a. WDK Station Original & Denoised Wind Speed. b. WDK Station Wind Speed Noise Density. c. Annual (West-East) Wind Speed Noise Residuals. d. Annual (North-South) Wind Speed Noise Residuals.

Table 3
Original relative and denoised optimal humidity concentration matrix.

Original data	Months	Weather stations	DIEP	KARI
		Probabilities	0.0809	0.9191
	Feb (2012)	0.9625	44.05	43.37
	Jun (2014)	0.0375	27.374	44.837
	Game value	43.4243		
Denoised data	Months	Weather stations	DIEP	KARI
		Probabilities	0.0777	0.9223
	Feb (2012)	0.9280	43.97	43.38
	Jun (2014)	0.0320	26.99	44.81
	Game value	43.4265		

Table 4
Original and denoised optimal temperature concentration matrix.

Original data	Months	Weather stations	CLA	WLO
		Probabilities	0.1794	0.8206
	Feb (2014)	0.6994	19.4	20.3
	Jan (2015)	0.3006	21.9	19.8
	Game value	20.1497		
Denoised data	Months	Weather stations	CLA	WLO
		Probabilities	**0.2803**	**0.7197**
	Dec (2012)	**0.0515**	22.40	19.0
	Feb (2014)	**0.9485**	19.80	20.0
	Game value	19.9749		

v. Eastern station of TSU has fairly spread noise pops with very few higher densities (See Table 1).
vi. The annual temperature noise residual plots reveal a relatively low and stable noise throughout the four years at Gell weather station

2.4. Summarised results for wind speed denoised data

As earlier noted, the wind speed data across the stations and over the given period were noticeably less noisy from the original data, hence the reason for chosing specifically the thresholding method of heuristic SURE-hard to provide the optimum denoising strategy. From the above simulations, we have the following summary (See Table 2):

i. WLO station in the West shows fairly spread with very few higher densities.
ii. ALEX in the North depicts bigger but fewer pops over some months but just fairly spread with a few higher densities.
iii. KARI in the South shows widely spread noise pops with higher densities
iv. TSU in the East has widely spread noise pops with higher densities.
v. WDK in the central region shows widely spread noise pops over the period with higher densities.

From the above simulations, the relevance of noise investigation in the Namibia's atmospheric data becomes very evident. According to Gelfan et al. (2015), 'the internal variability of the atmosphere ("climate noise"), is inherent to the climate system and considered as the lowest level of uncertainty achievable in climate impact studies', thus

Table 5
Original and denoised optimal wind speed concentration matrix.

Original data	Months	Weather stations	DIEP	WDK	ERICH	TSU	WAT
		Probabilities	0.2608	0.1875	0.2727	0.2420	0.0371
	Sep (2013)	0.0812	0.83	1.23	0.88	1.4	0.39
	Aug (2014)	0.1186	0.75	0.84	0.75	1.16	1.06
	Nov (2014)	0.0929	1.11	0.58	1.11	0.57	0.77
	Dec (2014)	0.0682	2.10	0.60	2.10	0.30	0.50
	Apr (2015)	0.6392	0.81	1.09	0.6	1.02	0.81
	Game value	1.1788					
Denoised data	Months	Weather stations	ERICH	WAT		TSU	
		Probabilities	0.4137	0.5171		0.0692	
	May (2012)	0.6486	0.79	1.08		1.07	
	Jan (2014)	0.2374	1.30	0.70		0.80	
	Dec (2014)	0.1139	1.20	0.80		0.77	
	Game Value	0.9572					

necessitating the need for increasing noise investigation in meteorological data of the country, especially in the climatic environments characterised by industrial pollutions.

3. Strategic weather stations with optimal meteorological factors

Employing game modelling approach and considering the months of the year and weather stations as the competitive parameters in a 2-player mixed strategy game, we define the expected value of the game as

$$MFS = \sum_{j=1}^{14}\sum_{i=1}^{48} f_{ij} m_i s_j \tag{7}$$

where

$s_j, j = 1, \ldots 14$ – the weather stations strategies.$m_i, i = 1, \ldots, 48$ – the months of the year strategies.f_{ij}, - the meteorological factor values.

In Eq. (7), M is the vector for the months of the year strategy, F, the factor's payoff matrix and S the station strategy vector.

In Tables 3–5 below, we have the results of comparing the original optimal game model values with the optimal denoised variants.

From Table 3 above, it is observed that both the optimally selected weather stations and months of the year are preserved with very significant small errors in the probabilities and the game values, (specifically of order 10^{-3} in the game values), thus depicting a high degree of "humidity noise stability" for the identified stations and the two optimal periods of the years. But as seen in Table 4 for the temperature optimal values, while the two stations were "temperature noise stable", however the optimal periods in the denoised case were rather different and, moreover, the optimal (game value) error was just of order 10^{-1}.

In Table 5, we observe significant but mixed wind speed noise instability, resulting in an exclusion of two weather stations (DIEP and WDK) and two years (2013 and 2015) in the optimal solutions. Three stations (ERICH, WAT and TSU) are however retained with year 2012 entering the optimal set and sharing the highest proportion of wind speed effect.

4. Conclusions

The exploratory study has clearly shown existence of some significant data (noise) pollution in the meteorological factors of the Namibia's atmospheric environments. The study specifically revealed higher humidity and temperature noise densities at the coastal (western) part of the country with the southern part being significantly humidity noisy, and the northern part similarly being temperature noisy. The central part of the country appeared to be less meteorologically noisy while the eastern part of the country is the least meteorologically noisy.

Identifying the parts of the country with noisy meteorologically data therefore provides very useful information about specific environments within the country that require quality meteorological data for useful planning. Thus, additional sources of data collection including in situ sources, especially for the meteorologically noisy parts of the country become very relevant for data validation. Since the accuracy, timing (sampling period), location and sampling method are evidently crucial, additional sources of defensible data that are most useful to make informed decisions that mitigate errors are also important in a comprehensive investigation of the Namibian atmospheric conditions.

This in turn suggests the need for further related investigation of various causes of data pollution in the meteorologically noisy parts of the country. A quick and intuitive example of such investigation is the study of the impact of industrial pollution on the meteorological data obtained from specific areas of the country.

Of great interest is the comparison of the optimal game model solutions for the original and denoised data. These specifically identified DIEP (slightly central) and KARI (in the south) as weather stations that are "humidity noise stable", and CLA (central) and WLO (in the west at the coast) as weather stations that are "temperature noise stable". Thus, these suggest that despite the noise in the respective meteorological data for these stations, the effects of the noise in the data appear to be manageable or optimally controllable, within the given period. Further investigation for different periods is therefore suggested to confirm long-term "meteorological noise stability" for these stations.

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Declarations of interest

None.

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